Test 3  
M1910, Spring 2010

1. Find the absolute minimum and maximum values of function $F$ on the given interval. Write your answers as ordered pairs.

$$F(x) = \frac{x^2 - 3}{x^2 - 1} \text{ on the interval } \left[\frac{3}{4}, \frac{1}{2}\right]$$

2. Find all values of $\theta$ on the interval $(0, 2\pi)$ for which the graph of $T(x) = \theta + 2 \cos \theta$ is increasing. Record your answer(s) using interval notation.

3. Sketch a graph of $y = \tan x$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Discuss the function’s concavity. Use calculus to support your statements.

4. Find all local (relative) minima and maxima of $p(x) = 8x^5 + 25x^4 - 20x^3 + 1$. Write your answers as ordered pairs.

5. Evaluate the limit, if the limit exists.

6. Determine whether the Mean Value Theorem can be applied to $f$ on the closed interval $[a, b]$. Support your conclusion. If the MVT can be applied, find all values of $c$ in the open interval $(a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

7. For some positive constant $C$, the temperature change $T$ in a patient is generated by a dose, $x$ of a drug given by

$$T(x) = \left(\frac{C}{2} - \frac{x}{3}\right)x^3.$$  

What dosage maximizes temperature change?

8. Find any critical numbers of the function $f$.

$$f(x) = \frac{3}{(5x^2 + 4x)^3}$$
9. Consider the graph below. Observe that the given line is tangent to $f$ at the point $(1, 4)$. Use differentials to approximate $f(0.9)$ and $f(1.05)$.

10. Find the points of inflection of the function $f$. Discuss the concavity of the graph.

$$f(x) = xe^{3x}$$